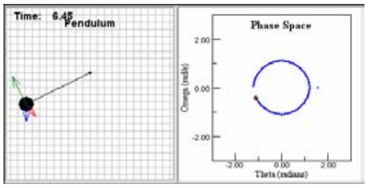
Worksheet for Exploration 16.4: Pendulum Motion, Forces, and Phase Space



A 1-kg pendulum bob is attached to a 9.8-m massless string to form a pendulum (**position is given in meters and time is given in seconds**). A graph of angular velocity **(rad/s)** vs. angle **(rad)** is shown. This graph is sometimes called a "phase space" representation of the motion. Restart. In addition,

- the red arrow represents the total force
- the blue arrow represents the force of gravity
- the green arrow represents the velocity

The phase-space representation of motion is just another way to describe an object's motion (like a position vs. time graph). For example, when would the phase-space representation of the motion be circular? Well, x and v would have to have the same frequency, be out of phase with each other by $\pi/2$ radians (or 90^{o}), and x_{max} and v_{max} would have to have the same magnitude. This occurs with simple harmonic motion when ω = 1rad/s.

You must first select the "drag pendulum" button, drag the pendulum bob into place, and then press "play" to begin the animation for a different initial angle.

Note that ω (1rad/s) referred to above is the angular frequency that describes how often each cycle is repeated. This number is constant for a simple harmonic oscillator. This is different than "omega" as listed in the graph. The graphed quantity refers to the rate at which real physical angle goes by as the pendulum moves along part of a circular path.

a. Given the information above and the information depicted in the animation, when does the pendulum motion approximate simple harmonic motion?
   i. You may need to ask your instructor to give you specific limits as to when a pendulum is undergoing simple harmonic motion.

b. Determine the maximum angle for approximate simple harmonic motion from the animation.

c. We have considered a special case of simple harmonic motion, ω = 1rad/s. What would the phase-space diagram look like for simple harmonic motion with a general ω?