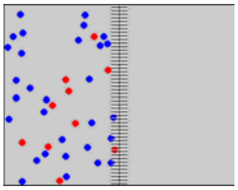
Worksheet for Exploration 21.3: Entropy, Probability, and Microstates



In the animation two containers are separated by a "membrane." Initially, no particles can cross the membrane. Note that the red and the blue particles are identical, they are colored so you can keep track of them. Once the particles are fairly evenly distributed in the left chamber, you are ready to let particles through. Try letting particles through the membrane. This animation allows about every other particle that hits the membrane to get through (equally in either direction). When there are about the same number of particles on both the left and right sides, pause the animation and count the number of red particles on each side and the number of blue particles on each side. Let the animation continue and stop it again a few seconds later when there are about the same number of particles on each side. Again, find the number of red and blue particles on each side. Restart.

a. Given that there are 30 blue particles and 10 red particles total, if you made many such measurements what would you expect the average number of red and blue particles to be on each side (when there are a total of 20 particles on each side)?

Red_{left}=________ Red_{right}=________

$Blue_{left}$=________ $Blue_{right}$=________

b. Now, restart the animation. Once the particles are fairly evenly distributed in the left chamber, try letting particles through the membrane a different way. Again, this animation allows about every other particle through the membrane.

c. When there are about the same number of particles on either side, count the red and blue particles on each side. What is different about the way this membrane is set up?
   i. You should make sufficient number of runs to record an average of each.

Red_{left}=________ Red_{right}=________

$Blue_{left}$=________ $Blue_{right}$=________

Discuss:

d. Is it possible for the first membrane to have this outcome?

Yes or No

e. Is this outcome likely?
   i. Discuss.

The reason that the second membrane does not appear "natural" is the second law of thermodynamics. One version of the second law is that the entropy of an isolated system always stays the same or increases (where entropy is defined as a measure of the disorder of the system). In other words, "natural" systems move in the direction of greater disorder. In the animations the first membrane seems "natural" because it allows for the most disorder—a random distribution of reds and blues on both sides. This is compared to the second membrane that only allows blue particles through, and thus the right side will always have only blue particles in it.

Another way to interpret the second law is in terms of probability. It is possible with the first animation to get 0 red particles in the right chamber, but it is very unlikely (just like it is possible you will win the lottery, but it is very unlikely). It is also possible for the second animation to behave as it does, but again, it is very unlikely. Consider the animation above with only six particles: four blue and two red. To keep track of things, we've colored the blue ones different shades of blue and the red ones different shades of red. Run the animation and notice how often there are three blue ones on the right side when there are three particles on each side. What follows will allow you to calculate the probability of this happening and show that when there are three particles on each side there is a 20% chance that there will be three blue ones in the right chamber.

f. Considering the different arrangements of three particles on each side, note that there are four different ways to get 3 blues on the right and two reds and one blue on the left (list these and click here to show them). Similarly, there are the same four ways to get 3 blue particles in the left chamber.
g. There are 6 ways to get the light red on the left and the dark red on the right with 2 blues each (click here to show these). Again, there are the same 6 ways to have the dark red on the left and the light red on the right.
h. This gives a total of how many different arrangements (of 3 particles on each side)? Since all these states are equally likely, you have only a 20% chance of having 3 blues in the right chamber.

As we add more particles, it becomes less likely to get all of one color on one side. With 40 particles, 30 blue and 10 red, there is only around a 0.02% chance that when there are 20 particles on each side, there will be 20 blue ones on the left and 10 red and 10 blue on the right. This is not impossible, but not very likely (better odds than your local lottery, where your odds might be around one in a couple of million). A more ordered state (20 blues on the right) is less likely, statistically, than a less ordered state (reds on both sides of the membrane, which is a more even mixing). Entropy is related to the number of available states that correspond to a given arrangement (mathematically, $S = k_B \ln W$, where S is entropy, W is the number of equivalent arrangements or microstates, and k_B is the Boltzmann constant).

Going back to our six particle example, there were more states that corresponded to one red in each chamber than two reds in one chamber, so one red in each chamber is a more likely state for the system to be in. Most of the time, however, we are dealing with more than six particles (usually around Avogadro's number), so the very ordered state is even less likely to happen. Connecting entropy to probability, then, gives a version of the second law that does not forbid the system from being in a highly ordered state; it simply says that it is highly unlikely.