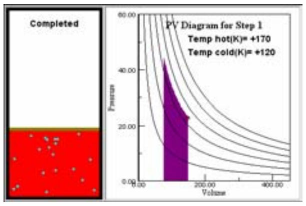
Worksheet for Exploration 21.1: Engine Efficiency



In this animation, N = nR (i.e., $k_B = 1$). This, then, gives the ideal gas law as PV = NT. Assume an ideal monatomic gas. The efficiency of an engine is defined as ε = (work out) / (heat in) = $|W| / |Q_H|$.

a. Pick a temperature for the hot reservoir (between 200 K and 150 K) and a lower temperature (between 150 K and 100 K) for the cold reservoir. (Note that new reservoir temperatures only register at the beginning of the engine cycle and that you should run the engine steps in order for the animation to make sense.) Find the work done for each step and the heat absorbed or released (remember that $\Delta U = (3/2)nR\Delta T = (3/2)N\Delta T$).

	W	Q_H	ΔU
Step 1			
Step 2			
Step 3			
Step 4			

b. Calculate the efficiency of the engine for these temperatures.

ε=________

c. Pick a different pair of temperatures for the reservoirs. Is this engine more or less efficient? (Calculate the efficiency for this new engine.)

Complete the table for the new temperatures.

	W	Q_H	ΔU
Step 1			
Step 2			
Step 3			
Step 4			

ε=________

d. Why is the engine in (c) more or less efficient?

e. What would make the engine even more efficient? Try it and explain.

f. Calculate the difference in temperatures of the reservoir divided by the temperature of the hot reservoir: $(T_H-T_L)/T_H = 1- T_L/T_H$. Compare this value with the efficiency number for each case above. For a Carnot engine, either calculation gives the efficiency because of the following:

ε_{carnot}=__________

g. For step 1: $W = Q_H = nRT_H\ln(V_1/V_0) = NT_H\ln(V_1/V_0)$, where V_0 is the volume at the beginning of step 1 and V_1 is the volume at the end of step 1. Explain why (do some algebra!) and verify with the animation.

h. Similarly, for step 3: $W = Q_L = nRT_L\ln(V_3/V_2) = NT_L\ln(V_3/V_2)$, so $|Q_L| = NT_L\ln(V_2/V_3)$, where V_2 is the volume at the beginning of step 3 and V_3 is the volume at the end of step 3. Explain why (do some algebra!) and verify with the animation.

i. Steps 2 & 4 are adiabatic, so $Q = 0$. From steps 2 & 4, $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ and $P_3V_3^{\gamma} = P_0V_0^{\gamma}$ where γ is the adiabatic coefficient (ratio of the specific heat at constant pressure to specific heat at constant volume). Using these relations and the ideal gas law, show (more algebra) that $(V_1/V_0) = (V_2/V_3)$.

j. Therefore, show for a Carnot engine $|W|/|Q_H| = 1 - |Q_L|/|Q_H| = 1 - T_L/T_H$.

The Carnot engine efficiency of $1 - T_L/T_H$ is the ideal efficiency for any engine operating between two reservoirs of T_H and T_L, because the net change in entropy is zero for a Carnot cycle (see Illustration 21.4). Often, you will compare the efficiency of other engines, $|W|/|Q_H|$, to the ideal Carnot engine efficiency. Note, that you cannot have a 100% efficient engine because that would require $T_L = 0$ (which is forbidden by the third law of thermodynamics). Another way to think about this is that to get an efficiency of 100% you would need to put the heat released into the low temperature reservoir back into an engine. But that engine would need to run between T_L and a lower temperature, and again you can't get to $T_L = 0$.