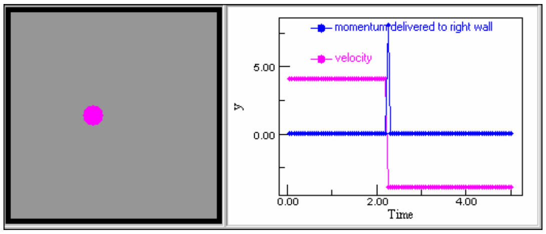
Worksheet for Exploration 20.1: Kinetic Theory, Microscopic and Macroscopic Connections



In this animation N = nR (i.e., $k_B = 1$). This, then, gives the ideal gas law as PV = NT. The average values shown, < >, are calculated over intervals of one time unit. Using the ideal gas law, we can make a connection between the macroscopic quantities of temperature (T) and pressure (P) and the individual microscopic properties of a particle of momentum (**p** = m**v**) and kinetic energy (1/2 mv^2). Restart.

Let's begin with one particle in an enclosed box and bouncing between two walls.

a. What is the change in momentum of the particle (use the velocity vs. time graph) as it hits the wall?
   i. Use information from the data table to determine the change in momentum.

Δp_{right}=_________

b. What is the average force that the right wall experiences over time? As a reminder, $\mathbf{F}_{avg} = \Delta\mathbf{p}/\Delta t$, so pick a time frame (like 20 or so), multiply the change in momentum each time by the number of collisions with the right wall, and then divide by the total time for this number of collisions.
   i. Another way to do this is to determine the time allocated for a single collision. Each collision here behaves identically. So you could simply measure the time between collisions, and use the momentum imparted for one collision. Averaging is easy when each event is the same.

$F_{avg\ right}$=_________

c. What is the average force on the left wall? The top and bottom walls?

$F_{avg\ left}$=_________

$F_{avg\ top\ or\ bottom}$=_________

$F_{avg\ either\ side}$=_________

d. Find the pressure on the surface of the box, Force/(area of all the walls). **The dimension of the wall into the screen is 1**.

e. Compare the pressure from (d) with the pressure on the box calculated from the ideal gas law and recorded in the table.

Increase the speed of the particle. The momentum delivered to the wall (and the force it experiences) will increase, thereby increasing the pressure. If the pressure of the gas increases (and if the volume is constant), the temperature of the gas will also increase.

f. What is the new speed of the particle?

g. What is the new pressure?

h. What is the new temperature?

By the same reasoning, increasing the mass of the particle will also increase the pressure, so the temperature should be connected to the mass of the particle as well. Increase the mass of the particle.

i. What is the new mass?

j. What is the new pressure?

k. What is the new temperature? The relation between the temperature and increased speed and mass is that the temperature is proportional to the kinetic energy.

One particle in an enclosed box is not very realistic, so let's add a second particle (of the same mass) with a different speed. This time, however, we'll plot the kinetic energy of each particle as a function of time and the change in momentum at any wall (an average of this will give us the pressure). The table now shows the average momentum change at te walls. How does this compare with the pressure you calculate using the ideal gas law?

l. The collision between the particles is an elastic one. How can you tell?

m. What is the connection between the temperature and the total kinetic energy?

Now let's add some more particles of the same mass and different speeds. The table gives the momentum delivered to the wall as particles collide with the wall (<d**p**/dt>), as well as the pressure calculated from the ideal gas law. This time we plot a histogram of the speeds of the particles. Stop the animation at some time and calculate the total kinetic energy of the ensemble of particles. This, divided by the number of particles, should be the same as the temperature of the system. This is the equipartition of energy theorem: The internal energy of a gas (the sum of the energy of all particles) is equal to $(f/2)k_BNT$, where f is the number of degrees of freedom for the atoms or molecules in a gas. In this case, the particles have 2 degrees of freedom, they can move in the x direction and y direction and thus $f = 2$. Because we are treating the gas particles as "hard spheres" (one of our assumptions in the ideal gas model), the internal energy of the gas is due to the kinetic energy of the particles and is equal to k_BNT and for this animation, $k_B = 1$.