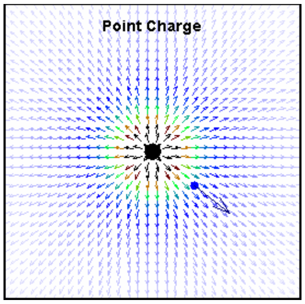
Worksheet for Exploration 24.2: Symmetry and Using Gauss's Law



Gauss's Law is always true: $\Phi = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A} = q_{\text{enclosed}}/\varepsilon_0$, but it isn't always useful for finding the electric field, which is what we are usually interested in. This should not be too surprising because to find **E**, using an equation like $\int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A} = q_{\text{enclosed}}/\varepsilon_0$, **E** has to be able to come out of the integral, and for that to happen, **E** needs to be constant on a surface. This is where symmetry comes in. Gauss's law is only useful for calculating electric fields when the symmetry is such that you can construct a Gaussian surface so that the electric field is constant over the surface and the angle between the electric field and the normal to the Gaussian surface does not vary over the surface **(position is given in meters and electric field strength is given in N/C)**. In practice, this means that you pick a Gaussian surface with the same symmetry as the charge distribution.

*Note some surfaces may be selected so that E is constant on part of the surface. If E changes on other parts of the surface that may be OK as long as the field direction is parallel to the surface. In other words, it is OK to include surfaces with non-constant E as long as the flux through those surfaces is zero at each point.

Consider a sphere around a point charge. The blue test charge shows the direction of the electric field. There is also a vector pointing in the direction of the surface normal to the sphere.

a. By moving the surface normal vector on the sphere and putting the test charge at three different points on the surface, find the value of $\mathbf{E} \cdot d\mathbf{A} =$ E dA cosθ (set dA = 1) at these three points (read the electric field values in the yellow text box). Are they the same? Why or why not?
   i. Note that you will need to be careful measuring the electric field strengths.

E_1=________ E_2=________ E_3=________

Same or not. Discuss.

Now, put a box around the same point charge. The test charge now shows the direction of the electric field, and the smallest angle between the vector and a vertical axis is shown (in degrees). The red vector points in the direction of the surface normal to the box (two sides show).

b. By moving the surface normal vectors on the box and putting the test charge at three different points on the top surface, find the value of $\mathbf{E} \cdot d\mathbf{A} =$ E dA cosθ (set dA = 1) at these three points. Are they the same? Why or why not?
   i. When you find E dA cos(θ) and set dA to 1, you have found the flux through a unit region of the surface. This is equivalent to finding the flux per unit area, or a “flux density”. The flux density may or may not be constant on given surfaces.

Flux Density$_1$=_______

Flux Density$_2$=_______

Flux $Density_3$=_______

c. In the context of your answers above, why is the sphere (as shown in the simulation) a better choice for using Gauss's law than the box?
   i. Also note whether the sphere can be a poor choice? Think about what you can do to make the flux density of the sphere non-uniform. Discuss or draw.

Let's try another charge configuration. Put a sphere around part of a charged plate (assume the gray circles you see are long rods of charge that extend into and out of the screen to create a charged plate that you see in cross-section).

**For parts d and e note that your book may discuss a large or "infinite" flat plate that extends to fill the entire plane containing the rods. Such a plate produces a uniform electric field on either side of the plate directed in or out depending on the sign of the charge. Here the simulation limits the charge locations to the simulation window and is NOT IDEAL like the situations in your book. Hence you will observe slight non-uniformities in the electric fields.

d. Would the value of $\mathbf{E} \cdot d\mathbf{A} = E\, dA \cos\theta$ be the same at any three points on the Gaussian surface?
   i. Again with dA =1 this is really a measure of flux density as above.

Flux $Density_1$=_______

Flux $Density_2$=_______

Flux $Density_3$=_______

e. Explain, then, why you would not want to use a sphere for this configuration.

Now, put a box around part of a charged plate (assume the points you see are long rods of charge that extend into and out of the screen to create a charged plate that you see in cross-section).

f. Find the value of **E** • d**A =** E dA cosθ at three points on the top. Are they essentially the same?
   i. Essentially the same in light of comments ** above.

Flux Density$_1$=_______

Flux Density$_2$=_______

Flux Density$_3$=_______

g. What about **E** • d**A =** E dA cosθ on the sides?
   i. Note earlier comments * which should apply to the sides of the box (4 sides). Discuss why the flux is “essentially” zero along each of these four sides.

For the plate, using a box as a Gaussian surface means that $\mathbf{E} \cdot d\mathbf{A} = E\, dA \cos\theta$ is a constant for each section (top, bottom and sides) and the electric field is a constant on the surface. This means you can write:

$$\int_{\mathbf{surface}} \mathbf{E} \cdot d\mathbf{A} = E \int_{\mathbf{surface}} dA = EA \text{ (for the surfaces where the flux is nonzero).}$$

h. Knowing that the charge per unit area on the big plate is σ, use Gauss's law to show that the expression for the electric field above or below a charged plate is $E = \sigma/2\varepsilon_0$ [SINGLE PLATE] and the direction of the electric field is away from the plate for a positively charged plate. In your textbook, you will probably also see an expression that says that the electric field is σ/ε_0 above or below the charged sheet. This holds true for conductors where σ is the charge / area on the top surface and there is the same amount of charge / area on the bottom surface (there is no net charge inside a conductor).

i. This second case where the result is $E = \sigma/\varepsilon_0$ is really two separate plates of charge. If the charges are the same then the region between the plates has a net field of zero, and outside as given which is due to contributions from both plates. A similar situation gives the field between two oppositely charged capacitor plates with the same formula. Sketch the two capacitor plates, the field for each plate, and then another sketch showing the total field in the region to the left, then between, then to the right of both plates.